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(6) THREE-DIMENSIONAL DIGITAL LINE SEGMENTS

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ABSTRACT

Digital arcs in 3D digital pictures are defined. The digital image of an arc is also defined. A digital arc is defined to be a digital line segment if it is the digital image of a line segment. It is shown that a digital line segment may be characterized by the chord property holding for its projections onto the coordinate planes. It is also shown that a digital line segment may not be characterized by its own chord property. A linear time algorithm is presented that determines whether or not a digital arc is a digital line segment.

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1. Introduction

Recent growing interest in three-dimensional (or simply, 3D) image processing [4,9,11,12,19,20] makes it essential to develop a theory of 3D discrete geometry. Some work has already been done on the subject [8,11].

The following two problems are among the many with which digital image processing is concerned. One is the problem of digitization, that is, how to represent continuous objects using finite sets of digital points. The other is the problem of retrieving from a set of digital points information about the object represented by it. Consider a line segment, for example. One problem is how to represent it by a set of digital points, and another is how we can tell whether or not a given set of digital points represents a line segment. Similar problems can be formulated for other types of curves.

Some of the 2D digital geometric properties that have been studied extensively are connectedness, straightness, and convexity [1,2,5,6,7,10,13,14,17]. Extensions to 3D digital geometry have been investigated for connectedness and convexity [16,8]. In this paper we study the property of straightness for sets of 3D digital points.

The straightness of 2D digital arcs was studied in [14]. It was shown that a 2D digital arc is straight (or equivalently, is a 2D digital line segment) if and only if it has the chord property. (This property is defined in the next section.) A

similar result is obtained for 3D digital arcs in this paper. We define how 3D arcs and curves are represented by sets of 3D digital points. Then we give a necessary and sufficient condition for a set of 3D digital points to represent a line segment. Next we present an algorithm that determines whether or not a given set of 3D digital points is straight (a 3D line segment).

In the next section we introduce terminology and definitions that are used throughout the paper. In particular, a scheme of digital representation of 3D arcs is introduced and the chord property is defined. In Section 3, we show that the chord property of the projections of a set of digital points characterizes the straightness of the set. It is also shown that the chord property of the set itself is not a necessary condition. Section 4 presents an efficient algorithm that determines the straightness of digital arcs and analyzes its time complexity. In the last section, we discuss the results obtained and their relation to those for sets of 2D digital points.

2. Definitions

The set of all points in 3D(2D) Euclidean space is denoted by E(E'). The set of all points in E(E') with integer-valued coordinates is denoted by D(D'). A point in D or D' is called a digital point.

Two points $d=(i,j,k)$ and $d'=(i',j',k')$ in D are called 26-neighbors if $\max\{|i-i'|,|j-j'|,|k-k'|\}=1$. Similarly, $d=(i,j)$ and $d'=(i',j')$ in D' are called 8-neighbors if $\max\{|i-i'|,|j-j'|\}=1$.

A 26-chain (8-chain) is a finite sequence of digital points in D(D') such that every element of the sequence except the first is a 26-neighbor (8-neighbor) of its predecessor. A set Q of digital points in D(D') is said to be 26-connected (8-connected) if for any two points d,d' of Q, there is a 26-chain (8-chain) in Q from d to d'.

The block distance between $\omega=(x,y,z)$ and $\omega'=(x',y',z')$ in E is defined by $\beta(\omega,\omega')=\max\{|x-x'|,|y-y'|,|z-z'|\}$. Similarly, the block distance beween $\omega=(x,y)$ and $\omega'=(x',y')$ in E' is defined by $\beta(\omega,\omega')=\max\{|x-x'|,|y-y'|\}$. Two points ω and ω' are said to be near each other if $\beta(\omega,\omega')<1$. Given a finite set Q of digital points, point ω is said to be near Q if there is a point d of Q to which ω is near.

(2D) digital arcs

A 26-(8-)connected finite subset R of D(D') is called a (2D) digital arc if every point of R except two has exactly two

26-(8-)neighbors in R and each of the exceptional two has exactly one 26-(8-)neighbor in R. A (2D) digital arc may be represented by a sequence $(d_0,d_1,\ldots,d_n)$ such that d_{i-1} is a neighbor of d_i for $1\le i\le n$.

(2D) simple digital curves

A 26-(8-)connected infinite subset C of D(D') is called a (2D) simple digital curve if every point of C has exactly two 26-neighbors (8-neighbors) in C. Hence a 26-connected (8-connected) finite subset of a (2D) simple digital curve is a (2D) digital arc.

If an arc f in E' crosses a coordinate line, there may be one or two digital points that are nearest to the crossing. When there are two, the one to the right of f with respect to its sense is chosen as the nearest. Suppose that an arc f in E crosses a coordinate plane, say the (z=k)-plane. There may be one, two or four digital points that are nearest to the crossing. Consider the case of two and let the crossing point be $(i+\frac{1}{2},y,k)$ $((x,j+\frac{1}{2},k))$, where $j\le y<j+\frac{1}{2}$ $(i\le x<i+\frac{1}{2})$ for some integer j(i). Let f_y (f_x) denote the projection of f onto the (y=j)-plane ((x=i)-plane). Then of the two points (i,j,k) and (i+1,j,k) ((i,j,k) and (i,j+1,k)), the one to the right of f_y (f_x) is chosen as the nearest to the crossing. In case of four-way ties, the digital point that lies to the right of both f_y and f_x is chosen as the nearest. When f crosses other coordinate planes, the nearest point to the crossing is determined similarly.

(2D) digital images of arcs and simple curves

Let f be an arc or a simple curve. Whenever f crosses a coordinate plane (line), the nearest digital point to the crossing becomes a point of the digital image of f, which is denoted by I(f)(I'(f)).

(2D) digital line segments

A (2D) digital arc is a (2D) digital line segment if there is a line segment f whose digital image is R, i.e., R=I(f) (R=I'(f)).

(2D) digital lines

A (2D) simple digital curve C is a (2D) digital line if every subset R of C that is a (2D) digital arc is a (2D) digital line segment.

We note that the (2D) digital image of an arc or a simple curve is not necessarily a (2D) digital arc or a (2D) simple digital curve. It was shown in [14] that the 2D digital image of a line segment in E' is a 2D digital arc. It will be shown later that the digital image of a line in E is a simple digital curve.

The following geometric property is useful in discussing and characterizing (2D) digital lines and line segments.

The chord property [14]

A set Q in D or D' is said to have the chord property if for any d,d' in Q, every point on $\overline{dd'}$ is near Q, where $\overline{dd'}$ is the line segment between d and d'.

3. Digital line segments

In this section we extend the results of [14] on 2D digital arcs to 3D digital arcs. Since they are used in the proofs below, we restate them as a lemma.

Lemma 1 (Theorems 1 and 2 in [14])

The 2D digital image of a line segment in E' is a 2D digital arc, and a 2D digital arc is a 2D digital line segment if and only if it has the chord property.

It is interesting to note that the 2D digital image of a line has the chord property but a 2D simple digital curve that has the chord property may not be the 2D digital image of any line. For example, consider the 2D simple digital curve $C=\{(i,0)|i$ is a negative integer$\} \cup \{(i,1)|i$ is a non-negative integer$\}$. It is easy to see that C has the chord property but there is no line whose 2D digital image is C. Thus, the chord property is a necessary and sufficient property for a 2D digital arc to have a line segment as a preimage but only a necessary condition for a 2D simple digital curve to have a line as a preimage. This is the reason for defining a 2D digital line not as having a line as its preimage but as satisfying the chord property.

Theorem 2

The digital image of a line is a simple digital curve.

Proof

Let a line f be given by $\frac{x-a}{\ell} = \frac{y-b}{m} = \frac{z-c}{n}$ and assume without

loss of generality that $\ell \geq m \geq n \geq 0$ and $\ell + m + n > 0$. Consider the line f_z which is the projection of f onto the (z=0)-plane. The 2D digital image of f_z on the plane is the projection on the plane of those digital points of I(f) that are obtained from the crossings of f on the (x=i)- and (y=j)-planes for all integers i and j. This is so because of the rules for obtaining the digital image of a line in E and the 2D digital image of a line in E'.

Suppose that f_z crosses the (y=j)-line at (x,j). If $i \leq x < i + \frac{1}{2}$ for some integer i, then (i,j) is a point of $I'(f_z)$. Also f_z crosses the (x=i)-line at (i,y), where $j \leq y < j - \frac{1}{2}$, since $\ell \geq m$. Thus, point (i,j) of $I'(f_z)$ is also obtained from the crossing of f_z on the (x=i)-line. If $i + \frac{1}{2} \leq x < i+1$, then (i+1,j) is a point of $I'(f_z)$, and f_z crosses the (x=i+1)-line at (i+1,y), where $j \leq y < j + \frac{1}{2}$, since $\ell \geq m$. Thus, the point (i+1,j) of $I'(f_z)$ is obtainable from the crossing of f_z on the (x=i+1)-line too. So every point of $I'(f_z)$ is obtainable from the crossings of f_z on the (x=i)-line for all integers i. Therefore, there is exactly one point of the 2D digital image of f_z on each (x=i)-line. Hence, if (i,j,k) and (i',j',k') are points of I(f) that are obtained from the crossings of f on the (x,z)- and (y,z)-planes, then j=j'. Also, if (i,j,k) and (i+1,j',k') are two such points, then $|j-j'| \leq 1$.

Now consider the line f_y which is the projection of f onto the the (y=0)-plane. By the same argument as above, we can show that if (i,j,k) and (i,j',k') of I(f) are obtained from the crossings

of f on the (x,y)- and (y,z)-plane, then $k=k'$. Also, if (i,j,k) and $(i+1,j',k')$ are two such points, then $|k-k'|\le 1$.

Therefore, there is exactly one point of $I(f)$ on the $(i=i)$-plane for each integer i, and if (i,j,k) and $(i+1,j',k')$ are points of $I(f)$, then they are 26-neighbors. Thus, $I(f)$ is a simple digital curve. □

Corollary 3

The digital image of a line segment is a digital arc. □

Given a set Q of digital points, let Q_t denote the projection of Q onto the $(t=0)$-plane for $t=x,y$ and z. In the following theorem, we assume without loss of generality that if $d_1=(i_1,j_1,k_1)$ and $d_2=(i_2,j_2,k_2)$ are the endpoints of a digital arc R, then $i_2-i_1\ge j_2-j_1\ge k_2-k_1$.

Theorem 4.

A digital arc R is a digital line segment if and only if both R_y and R_z are 2D digital arcs and have the chord property.

Proof

Suppose that the digital arc R is a digital line segment, that is, $R=I(f)$ for some digital line segment f. Suppose that f is given by $\frac{x-a}{\ell} = \frac{y-b}{m} = \frac{z-c}{n}$ for $u\le x\le v$. Since $i_2-i_1\ge j_2-j_1\ge k_2-k_1$, we may assume with no loss of generality that $\ell\ge m\ge n\ge o$. Consider the line segment f_z which is the projection of f onto the $(z=0)$-plane. It is obvious from the proof of Theorem 2 that $I'(f_z)$ is exactly R_z and is a 2D digital arc. Since R_z is a 2D digital image of a line segment, it has the chord property by Lemma 1. Similarly, R_y also is a 2D digital arc and has the chord property.

Now suppose that both R_y and R_z are 2D digital arcs and have the chord property. By Lemma 1, there exist line segments f_y and f_z such that $R_y=I'(f_y)$ and $R_z=I'(f_z)$. Let f_y and f_z be given by $\frac{x-a}{\ell} = \frac{z-c}{n}$ and $\frac{x-a}{\ell} = \frac{y-b}{m}$ for $u \le x \le v$, respectively. Consider the line segment $\frac{x-a}{\ell} = \frac{y-b}{m} = \frac{z-c}{n}$ for $u \le x \le v$. Since $i_2-i_1 \ge j_2-j_1 \ge k_2-k_1 \ge 0$, we may assume that $\ell \ge m \ge n \ge 0$. Let (i,j,k) be a point of R. Then its projection onto the $(z=0)$-plane is (i,j) and (i,j) is a point of $I'(f_z)$. Thus, it is the nearest digital point to the crossing of f_z on the $(x=i)$-line. Similarly, point (i,k) is the projection of (i,j,k) onto the $(y=0)$-plane and a point of $I'(f_y)$, and so is the nearest digital point to the crossing of f_y on the $(x=i)$-line. Therefore, (i,j,k) is the nearest point to the crossing of f on the $(x=i)$-plane. So $R \subseteq I(f)$. Obviously, from the proof of Theorem 2, the projections of R onto R_y and R_z are one-to-one. Also, $I'(f_y) \subseteq R_y$ and $I'(f_z) \subseteq R_z$. So $I(f) \subseteq R$. Therefore, $R=I(f)$ and R is a digital line segment. $\square$

<u>Corollary 5</u>

A simple digital curve C is a digital line if and only if it has the chord property.

<u>Proof</u>

Suppose that C has the chord property. Let R be any subset that is a digital arc. Since C has the chord property, so does R. Hence, R is a digital line segment by Theorem 4, so by definition C is a digital line.

Now suppose that C is a digital line. Let d_1, d_2 be any two points of C and R the 8-connected subset of C whose endpoints are d_1 and d_2. Then by definition, R is a digital line segment and has the chord property by the above theorem. So $\overline{d_1 d_2}$ is near R and thus near C. Hence, C has the chord property. □

Again, as in the case of 2D simple digital curves, a simple digital curve which has the chord property may not have any line as a preimage.

It is easy to see that if a digital arc R has the chord property, then at least two of R_x, R_y and R_z are 2D digital arcs. Obviously R_x, R_y and R_z have the chord property. So R is a digital line segment. Thus, the chord property is sufficient for a digital arc to be a digital line segment. However, it is not necessary. As an example, consider the

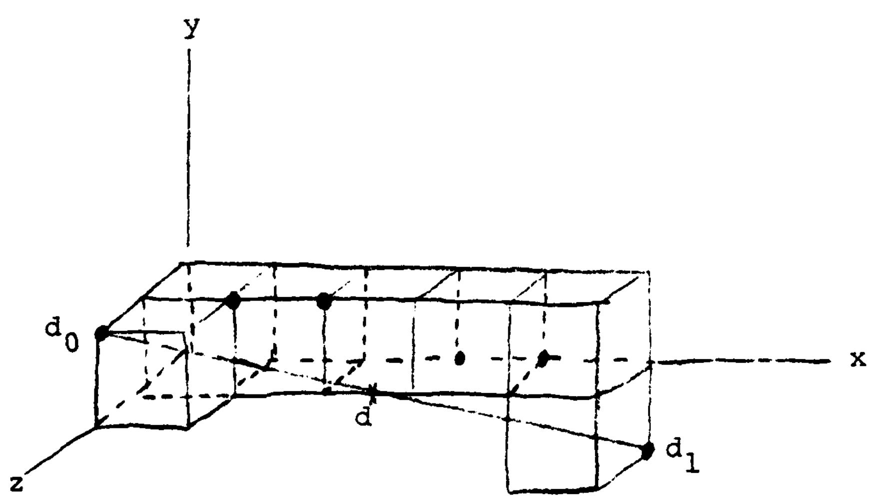

Figure 1

digital arc $R=\{(0,1,2),(1,1,1),(2,1,1),(3,0,0),(4,0,0),(5,-1,0)\}$ as shown in Figure 1. Then $R_y=\{(0,1),(1,1),(2,1),(3,0),(4,0),(5,-1)\}$ and $R_z=\{(0,2),(1,1),(2,1),(3,0),(4,0),(5,0)\}$ are 2D digital arcs and have the chord property. Thus, R is a digital line segment. Now consider points $d_0=(0,1,2)$ and $d_1=(5,-1,0)$ of R. The point $(2\frac{1}{2},0,1)$ is on $\overline{d_0 d_1}$ but it is not near R, so R does not have the chord property.

4. Algorithm

We present an efficient algorithm to determine whether or not a given digital arc is a digital line segment. We assume that digital arcs are contained in a cube of size N, that is, if $d=(i,j,k)$ is a point of a digital arc, then $0 \le i,j,k \le N$.

A digital arc can be represented by a chain code starting with one of its endpoints [15]. Since the chain code of a digital arc can be transformed into the sequence of digital points of the arc on-line in linear time, we assume that an arc is represented by a sequence of digital points $(d_0, d_1, \ldots, d_n)$, where $d_\ell = (i_\ell, j_\ell, k_\ell)$ is a 26-neighbor of $d_{\ell-1}$ for all $0 < \ell \le n$. We may also assume without loss of generality that $i_n - i_0 \ge j_n - j_0 \ge k_n - k_0$.

The algorithm is based on Theorem 4 and on results in [6,7], which are summarized as a lemma below.

Lemma 6 (Theorem 7 in [7] and Lemma 10 in [6])

A 6-connected finite set Q of digital points has the chord property if and only if H(Q), the convex hull of Q, does not contain any point of $\bar{Q}$.

The "if" part of Theorem 4 together with Lemma 6 yields a straightforward algorithm. That is, given a digital arc R, construct R_x and R_y and check if both are 2D digital arcs. If not, R is not a digital line segment. If so, check if the convex hull $H(R_y)$ or $H(R_z)$ contains any point of $\bar{R}_y$ or of $\bar{R}_z$,

respectively. If so, R is not a digital line segment, and if not, R is a digital line segment. However, the algorithm is not efficient. For, R_y and R_z may contain $O(N^2)$ elements and the construction of $H(R_y)$ and $H(R_z)$ has worst-case time complexity $O(N^2 \log N)$ [3]. Thus, the time complexity of the algorithm is at least $O(N^2 \log N)$.

To develop a more efficient algorithm, we note a simple fact that enables us to eliminate in O(N) time all digital arcs that consist of more than N points. For the digital arc R described above to be a digital line segment, we must have $i_\ell+1=i_{\ell+1}$ for each ℓ, $1\le\ell\le n$. Since $0\le i\le N$ for any point (i,j,k) of R, any digital arc that consists of more than N points is not a digital line segment. Thus, after this elimination step, we need to examine only the digital arcs that consist of no more than N elements.

Now, given a 6-connected set Q of n digital points, the algorithm CONVEX in [6] constructs the convex hull H(Q) of Q and checks whether H(Q) contains any point of $\bar{Q}$ in O(n) time. Since R contains at most N points, constructing and checking $H(R_y)$ and $H(R_z)$ takes $O(\min\{n,N\})$ time.

We briefly describe how the algorithm CONVEX is applied to 2D digital arcs. Given a 2D digital arc R represented by a sequence $(d_0,d_1,\ldots,d_n)$, where $d_\ell=(i_\ell,j_\ell)$, suppose that $i_\ell+1=i_{\ell+1}$ for all $1\le\ell\le n$. A subsequence $(d_s,d_{s+1},\ldots,d_t)$ is called a run if $j_s=j_{s+1}=\ldots=j_t$, $j_{s-1}\ne j_s$ and $j_t\ne j_{t+1}$. We call d_s a

left corner point of R and d_t a right corner point of R. In case of a run of one point, the point is both the left and right corner point. For any left corner point $d_s=(i_s,j_s)$ of R, (i_{s-1},j_s) is a point of $\bar{R}$ and is called a left corner point of $\bar{R}$. Similarly, if $d_t=(i_t,j_t)$ is a right corner point, (i_{t+1},j_t) is a point of $\bar{R}$ and is called a right corner point of $\bar{R}$.

While traversing clockwise the right corner points and then the left corner points of R in successive runs of R, we build a sequence P of the corner points of R. Similarly, we build a sequence $\bar{P}$ of the corner points of $\bar{R}$. We apply the convex hull construction algorithm in [3,18] to P to obtain the convex hull H(R) of R. We then check whether any of the points of $\bar{P}$ is a point of H(R). If so, R does not have the chord property and is not a 2D digital line segment. Otherwise, R has the chord property and hence is a 2D digital line segment.

<u>Algorithm</u> DIGITAL-LINE-SEGMENT(R,n)

//R is a 2D digital arc represented by a sequence of n+1 points.//

Step 1. For each $\ell=0,1,\ldots,n-1$ <u>do</u>

check if $i_\ell+1=i_{\ell+1}$

<u>if</u> not <u>then</u> return (<u>FALSE</u>); stop.

Step 2. Build R_y and R_z, the projections of R onto the (y=0)-plane and (z=0)-plane, respectively.

Step 3. Construct P_y and $\bar{P}_y$, the sequences of corner points of R_y and $\bar{R}_y$, respectively.

Step 4. Construct $H(P_y)$, the convex hull of P_y, which is in fact the convex hull of R_y.

Step 5. <u>If</u> $H(P_y)$ contains a point of $\bar{P}_y$ <u>then</u> return (<u>FALSE</u>); stop.

Step 6. Repeat steps 3-5 for R_z.

Step 7. Return (<u>TRUE</u>); stop.

<u>Theorem 7</u>

Given a digital arc represented by a sequence, algorithm DIGITAL-LINE-SEGMENT determines whether or not the digital arc is a digital line segment and its time complexity is $O(\min\{n,N\})$. □

5. Discussion

We have defined the digital images of arcs and curves and the notion of 3D digital arcs and curves. We have shown that a digital arc is a digital line segment if and only if two of its projections onto the (x=0)-, (y=0)- and (z=0)-planes are 2D digital line segments. The third projection may not even be a 2D digital arc. Still, this closely corresponds to the case of (continuous) arcs in 3D Euclidean space, where an arc is a line segment if and only if two, in fact all three, of its projections onto the coordinate planes are line segments.

In [14] it was shown that a set of 2D digital points is digitally convex if and only if it has the chord property. Thus, a 2D digital arc is a 2D digital line segment if and only if it is digitally convex. If "digital" and "digitally" are deleted from this statement, we obtain a true statement about 2D line segments. Thus, 2D line segments and 2D digital line segments both have the important geometric property of being convex.

However, the same does not hold in the 3D case. It was proved in [8] that the chord property is a necessary but not sufficient condition for a set of 3D digital points to be digitally convex. In Section 3, we showed that the chord property is a sufficient but not necessary condition for a 3D digital arc to be a 3D digital line segment. Thus, it is

not true that a 3D digital arc is a 3D line segment if and only if it is digitally convex, but it is true for 3D arcs. This is one of many examples that show the nontriviality of extending 2D digital geometry to 3D digital geometry.

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Digital arcs in 3D digital pictures are defined. The digital image of an arc is also defined. A digital arc is defined to a digital line segment if it is the digital image of a line segment. It is shown that a digital line segment may be characterized by the chord property holding for its projections onto the coordinate planes. It is also shown that a digital line segment may not be characterized by its own chord property. A linear time algorithm is presented that determines whether or not a digital arc is a digital

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line segment.